Entropy function and higher derivative corrections to entropies in (anti-)de Sitter space

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ABSTRACT: We first briefly discuss the relation between black hole thermodynamics and the entropy function formalism. We find that an equation which governs the relationship between Sen's entropy function and black hole entropy, can quickly give higher order corrections to entropy of pure (anti-) de Sitter space without knowing the corrected metric. We also show that near horizon geometry and the entropy function extremization is no longer required for pure (anti-)de Sitter space. The entropy of (anti-)de Sitter space and Schwarzschild-(anti-) de Sitter black holes together with Gauss-Bonnet terms, R^2 terms **and** R^4 terms are calculated as concrete examples.

KEYWORDS: Black Holes in String Theory, Black Holes, Classical Theories of Gravity.

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Contents

1. Introduction

Recently, Sen in ref. [1] introduced the entropy function method to calculate the entropy of extremal black holes with near horizon geometry $AdS_2 \times S^{n-2}$ by defining the entropy of the extremal black hole to be the extremal limit of the entropy of a non-extremal black hole so that one can use the Wald's formula for entropy given in [2, 3]. The entropy function method is an useful approach for computing the entropy from the Wald formula and it has been generalized to many solutions in supergravity theory no matter extremal or non-extremal solutions [4 – 29]. Actually, the entropy formula of Wald is based on the first law of black hole mechanics and it might not work for extremal black holes because of the vanishing surface gravity and vanishing bifurcation surface. On the other hand, the entropy function method works for extremal black holes with near horizon geometry $AdS_2 \times S^{n-2}$, and it might be difficult to apply this method to non-extremal black holes or more general conditions. This is because there is a strong hypothesis for the entropy function method [30]: *in any general covariant theory of gravity coupled to matter fields, the near horizon geometry of a spherically symmetric extremal black hole in n dimensions has*

$SO(2,1) \times SO(D-1)$ *isometry*. Apparently, the near horizon geometry of a non-extremal black hole might not satisfy the above argument.

The purpose of this paper is to compute the entropy in (anti-)de Sitter (dS) space with higher derivative gravity terms. Basing on the method developed by Cai and Cao [29], in this paper, we will show that a simple integration formula can quickly give the entropy of neutral black holes in dS and AdS spacetime, including the higher derivative corrections to the entropy. Black holes in dS and AdS sapcetimes are extremely important in many aspects [31]. They have special interest from the holographic point of view due to the well known AdS/CFT correspondence (dS/CFT correspondence) [32, 33]. The Hawking-Page transition for Schwarzschild-AdS (SAdS) black holes plays an important role in AdS/CFT correspondence where it was interpreted by Witten [32] as the confinement-deconfinement transition in dual gauge theory. In this sense, Schwarzschild-AdS is the important tool to describe thermodynamics of CFT and give crucial support of AdS/CFT correspondence.

Before going to concrete computation, we first summarize the differences between the method developed by Sen and the method to be used in this work. In general, the Sen's entropy function method is composed of the following three arguments [1]:

1) For an extremal charged black hole with near horizon geometry $AdS_2 \times S^{n-2}$. The part of AdS_2 metric is deformed into $v_1(-r^2 dt^2 + dr^2/r^2)$, while the S^{n-2} part of the metric has the form $v_2 d\Omega_{n-2}^2$. v_1 and v_2 are regarded as constant value here. The deformed metric is assumed as a solution of the equation of motion for a special action, where gravity is coupled to a set of electric and magnetic fields and neutral scalar fields u_s.

2) Define an entropy function over the horizon S^{n-2}, which is a function of v_i and u_s, and the electric and magnetic field (e_i, p_i).

3) One can find that for given e_i, p_i, the values u_s of the scalar fields as well as the sizes v_1 and v_2 of AdS_2 and S^{n-2} are determined by extremizing the entropy function with respect to the variables u_s, v_1 and v_2. The entropy function at its extremum point is proportional to the entropy of black holes.

In the following, we can see that for dS and AdS spacetimes:

(i) Extremizing the entropy function is not required. And also, we need not deform the metric into the near horizon geometry $AdS_2 \times S^{n-2}$. The configurations of spacetime can be used directly without any change in calculating the entropy. The near horizon limit is also unnecessary in obtaining dS entropies including higher derivative gravity terms.

(ii) We do not need to assume the radii of dS and AdS space as some constants, such as v_1 and v_2. (One may simply assume these parameters as 1. Actually $v1$ and $v2$ are quite trivial in calculating extremal stringy black hole entropy, because in the end their values are always 1, for example, see [16, 28])

(iii) Especially for pure dS and AdS space, one can still obtain the modified entropy formula without knowing the solutions of higher derivative gravity.

The organization of this paper is as follows. In next section we give a brief review of previous work on Noetherian entropy and entropy function. Some useful formulae are derived from black hole thermodynamics. Section 3 discusses the entropy of pure dS space using our new method. DS entropy with Gauss-Bonnet correction is calculated in section 4, where we compare our result with the entropy obtained by Wald's formula. In section 5 we deal with the dS entropy in R^2 gravity theory. The entropy of black holes in AdS is investigated in section 6. The R^4 correction to the entropy is included in this section. The last section contains our main conclusions and discussions.

2. Brief review of Noetherian entropy and entropy function

In this section, we will briefly review some important results made in the previous work [1, 2, 29], following the framework of Lagrangian field theories developed by Wald and viewing the Lagrangian as an n-form $\mathbf{L}(\psi)$, where $\psi = \{g_{ab}, R_{abcd}, \Phi_s, F_{ab}^I\}$ denotes the dynamical fields considered in this paper, including the spacetime metric g_{ab}, the corresponding Riemann tensor R_{abcd}, the scalar fields $\{\Phi_s, s = 0, 1, \cdots\}$, and the U(1) gauge fields $F_{ab}^I = \partial_a A_b^I - \partial_b A_a^I$ with the corresponding potentials $\{A_a^I, I = 1, \cdots\}$. Under this definition, the variation of $\mathbf{L}$ is

$$\delta\mathbf{L} = \mathbf{E}_\psi \delta\psi + d\mathbf{\Theta}, \tag{2.1}$$

where $\mathbf{\Theta}$ is an $(n-1)$-form, which is called *symplectic potential form*, $\mathbf{E}_\psi$ corresponds to the equations of motion for the metric and other fields. Let ξ be any smooth vector field on the space-time manifold, then one can define a *Noether current form* as

$$\mathbf{J}[\xi] = \mathbf{\Theta}(\psi, \mathcal{L}_\xi \psi) - \xi \cdot \mathbf{L}. \tag{2.2}$$

The fact that $d\mathbf{J}[\xi] = 0$ will be preserved when the equations of motion are satisfied shows that a locally constructed $(n-2)$-form $\mathbf{Q}[\xi]$ can be introduced and an "on shell" formula can be obtained

$$\mathbf{J}[\xi] = d\mathbf{Q}[\xi]. \tag{2.3}$$

Wald's analysis based on the first law of black hole thermodynamics showed that for general stationary black holes, the black hole entropy is a kind of Noether charge at horizon [2] and can be expressed as

$$S_{\text{BH}} = 2\pi \int_{\mathcal{H}} \mathbf{Q}[\xi], \tag{2.4}$$

where ξ represents the Killing field on the horizon, and $\mathcal{H}$ is the bifurcation surface of the horizon. It should be noted that the Killing vector field has been normalized to have unit surface gravity.

In fact, we can even go further, the definition of the Noether charge $\mathbf{Q}[\xi]$ can be extended in an arbitrary manner to ψ which do not satisfy the equations of motion. By contrast, we call this the so-called "off shell" form of the Noether charge, which is defined by the relation [34]

$$\mathbf{J}[\xi] = d\mathbf{Q}[\xi] + \xi^a \mathbf{C}_a, \tag{2.5}$$

where $\mathbf{C}_a$ is locally constructed out of the dynamical fields in a covariant manner and $\mathbf{C}_a = 0$ reduces to the previous definition (2.3) "on shell". Noether charge defined in (2.5) can be written as

$$\mathbf{Q} = \mathbf{Q}^F + \mathbf{Q}^g + \cdots \tag{2.6}$$

with

$$\mathbf{Q}^F_{a_1 \cdots a_{n-2}} = \frac{\partial \mathcal{L}}{\partial F^I_{ab}} \xi^c A^I_c \boldsymbol{\epsilon}_{aba_1 \cdots a_{n-2}}, \tag{2.7}$$

$$\mathbf{Q}^g_{a_1 \cdots a_{n-2}} = -\frac{\partial \mathcal{L}}{\partial R_{abcd}} \nabla_{[c} \xi_{d]} \boldsymbol{\epsilon}_{aba_1 \cdots a_{n-2}}. \tag{2.8}$$

The "$\cdots$" terms are not important for our following discussion, so we brutally drop them at first. The relevant discussion can be found in a recent paper [29].

On the other hand, A. Sen observed that the entropy of a kind of extremal black holes which have the near horizon geometry $AdS_2 \times S^{n-2}$ can be obtained by extremizing the so-called "entropy function" $\mathbf{f}$ with respect to the moduli on the horizon [1]

$$S_{\text{BH}} = 2\pi \mathbf{f} = 2\pi \left(e_i q_i - f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) \right). \tag{2.9}$$

where f is defined by

$$f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) = \int dx^2 \cdots dx^{n-1} \sqrt{-detg} \mathcal{L}. \tag{2.10}$$

Here $\sqrt{-detg} \mathcal{L}$ is the Lagrangian density, expressed as a function of the metric $g_{\mu\nu}$, the scalar fields ϕ_s, the gauge field strength $F^{(i)}_{\mu\nu}$ and covariant derivatives of these fields.

It was shown in [1] that if we denote by $\mathcal{L}_\lambda$ a deformation of $\mathcal{L}$ in which we rescale all factors of Riemann tensor $R_{\alpha\beta\gamma\delta}$ by $\lambda R_{\alpha\beta\gamma\delta}$ and define on the near horizon geometry

$$f_\lambda = \sqrt{-det\, g} \mathcal{L}_\lambda, \tag{2.11}$$

the following relation may be found

$$\left. \frac{\partial f_\lambda}{\partial \lambda} \right|_{\lambda=1} = \int_{\mathcal{H}} \sqrt{-det\, g} R_{\alpha\beta\gamma\delta} \frac{\partial \mathcal{L}}{\partial R_{\alpha\beta\gamma\delta}} dx^1 \cdots dx^{n-2} = f - e_i \frac{\partial f}{\partial e_i}, \tag{2.12}$$

where $\alpha, \beta, \gamma, \delta$ are summed over the coordinates r and t.

Now following our previous work on black hole thermodynamics and entropy function [35] we give an alternative way to calculate the black hole entropy. We consider an n-dimensional spherically symmetric black hole with the metric in the form of

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{a^2(r)} + b^2(r)d\Omega^2_{n-2}, \tag{2.13}$$

where a and b are functions of r, and $d\Omega^2_{n-2}$ is the line element for S^{n-2}. Since $\boldsymbol{\Theta} = 0$ if ξ is a Killing vector, we find by integrating over a Cauchy surface $\mathcal{C}$ on eq. (2.3)

$$\int_{\mathcal{C}} \mathbf{J} = -\int_{\mathcal{C}} \xi \cdot \mathbf{L} = \int_{\mathcal{C}} d\mathbf{Q}[\xi] = \int_{\infty} \mathbf{Q} - \int_{\mathcal{H}} \mathbf{Q} \tag{2.14}$$

where $\mathcal{H}$ denotes the interior boundary, and we have used the Stokes theorem. For an asymptotically flat, static spherically symmetric black hole, one can simply choose $\xi = \partial_t = \frac{\partial}{\partial t}$, then the free energy of the system is shown to be [35]

$$F = \mathcal{E} - \int_{\mathcal{H}} \mathbf{Q}[\xi^a], \tag{2.15}$$

where $F = T I_E$ with T and I_E the temperature and Euclidean action respectively. $\mathcal{E}$ in above formula is the "canonical energy", which is defined by [2],

$$\mathcal{E} = \int_{\infty} (\mathbf{Q}[t] - t \cdot \mathbf{B}), \tag{2.16}$$

where $\mathbf{B}$ is an $(n-1)$-form given by

$$\delta \int_{\infty} t \cdot \mathbf{B} = \int_{\infty} t \cdot \mathbf{\Theta}.$$

The variation of eq. (2.15) leads to

$$\delta F = \delta \mathcal{E} - \delta \int_{\mathcal{H}} \mathbf{Q}[\xi^a] \tag{2.17}$$

Now by noting that the Noetherian charge can be decomposed into two parts as shown in (2.6), we consider a stretched region near the horizon ranged from r_H to $r_H + \delta r$,

$$\delta \int_{\mathcal{H}} \mathbf{Q}[\xi] = \int_{r_H}^{r_H + \delta r} \left(\mathbf{Q}^{\mathrm{F}}[\xi] + \mathbf{Q}^g[\xi] \right) \tag{2.18}$$

The Killing equation gives $\nabla_{[a}\xi_{b]} = 2\kappa\epsilon_{ab}$ (where κ is the surface gravity of the hole), and the two parts are found to be [36]

$$\int_{r_H}^{r_H + \delta r} \mathbf{Q}^g[\partial_t] = \delta r \left[\kappa' E + \kappa E' \right]_{r_H} + \mathcal{O}(\delta r^2), \tag{2.19}$$

$$\int_{r_H}^{r_H + \delta r} \mathbf{Q}^F[\partial_t] = q_I e_I \delta r + e_I q_I' \delta r + \mathcal{O}(\delta r^2). \tag{2.20}$$

where $E(r)$ is defined as

$$E(r) \equiv - \int_{\mathcal{H}} \frac{\partial \mathcal{L}}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} dx^1 \cdots dx^{n-2}. \tag{2.21}$$

The above formula is exactly the Wald formula for entropy without the factor 2π [2]. Therefore, we find $E(r)$ is related to the entropy by $2\pi E(r_H) = S$. e_I and the U(1) electrical-like charges in eq. (2.20) are defined to be

$$e_I \equiv F_{rt}^I(r_H), \tag{2.22}$$

$$q_I \equiv \int_r \frac{1}{(n-2)!} \frac{\partial \mathcal{L}}{\partial F_{ab}^I} \epsilon_{aba_1 \cdots a_{n-2}} dx^{a_1} \wedge \cdots \wedge dx^{a_{n-2}}$$

$$= \frac{\partial}{\partial e_I} \int_{r_H} \frac{\mathcal{L}}{2(n-2)!} \epsilon^{ab} \epsilon_{aba_1 \cdots a_{n-2}} dx^{a_1} \wedge \cdots \wedge dx^{a_{n-2}} = \frac{\partial f(r_H)}{\partial e_I}, \tag{2.23}$$

where we have written $F_{ab}^I(r_H)$ as $e_I \epsilon_{ab}$. If the near horizon extension $r_H \to r_H + \delta r$ is also done for the free energy, we find that

$$\delta F = -\int_{r_H}^{r_H+\delta r} f dr = -f(r_H)\delta r + \mathcal{O}(\delta r^2) \tag{2.24}$$

Substituting eqs. (2.19), (2.20) and (2.24) into eq. (2.17), we obtain

$$\mathbf{f}\delta r = -S\delta T, \tag{2.25}$$

where $\mathbf{f} = (-f(r_H) + q_I e_I)$ is the entropy function for extremal black hole as shown in (2.9), and we have used the relation $\delta\mathcal{E} = T\delta S - e_I q_I' \delta r$.[1] In the limit $\delta r \to 0$, we obtain an equation which governs the entropy function for non-extremal black holes

$$ST' = -\mathbf{f}, \tag{2.26}$$

where prime denotes derivative with respect to r.

What we discussed above is the asymptotically flat case. The non-asymptotically flat case, such as asymptotically dS or AdS cases, however, is proved to be a little different from the one discussed above due to the definition of the Hamiltonian. By showing the difference we will start with looking at the Noetherian definition of mass in AdS spacetime. The definition of Hamiltonian is shown to be [37]

$$\delta H = \int_{\hat{R}} (\delta \mathbf{Q}[\xi] - \xi \cdot \mathbf{B}) - \int_{\hat{R}} \delta(\mathbf{Q}_{\text{AdS}}[\tilde{\xi}] - \tilde{\xi} \cdot \mathbf{B}_{\text{AdS}}), \tag{2.27}$$

where $\hat{R}$ is a cutoff at an outer boundary. This is unlike the flat case where the integration occurs at infinity. The second term in (2.27) corresponds to a constant by which we preserve the fact that the energy is zero in pure AdS. Notice that ξ^a is fixed during the variation, the spacetime and AdS background should have the same boundary geometry at $r = \hat{R}$, which leads to $|\tilde{\xi}|^2 = |\xi|^2$ on the boundary. In the end, taking into account the linear relation between $Q[\xi]$ and ξ one obtains the mass of the system [37]

$$\mathcal{E} = \int_{\hat{R}} (\mathbf{Q}[t] - t \cdot \mathbf{B}) - \left[\left(\frac{g_{tt}}{g_{tt}^{\text{AdS}}} \right)^{1/2} \right]_{r=\hat{R}} \int_{\hat{R}} (\mathbf{Q}_{\text{AdS}}[t] - t \cdot \mathbf{B}_{\text{AdS}}), \tag{2.28}$$

where we have taken ξ to be the time translation Killing vector since we only consider the static case.

We know that ξ vanishes on the bifurcate horizon hence $\mathbf{\Theta} = 0$ and the Noether current simplifies to $\mathbf{J} = d\mathbf{Q} = -\xi \cdot \mathbf{L}$. Integrating it over a Cauchy surface $\mathcal{C}$ with the interior boundary $\mathcal{H}$ as the event horizon of the black hole, and the outer boundary at $r = \hat{R}$, we get

$$\int_{\mathcal{H}} \mathbf{Q}[t] = \mathcal{E} + \int_{\mathcal{C}} \xi^t \cdot \mathbf{L}(g_{\text{BH}}) + \int_{\hat{R}} t \cdot \mathbf{B} + \left[\left\{ \frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}} \right\}^{1/2} \right]_{r=\hat{R}} \int_{\hat{R}} (\mathbf{Q}_{\text{AdS}}[t] - t \cdot \mathbf{B}_{\text{AdS}}). \tag{2.29}$$

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[1]More general form of the first law of black hole thermodynamics is $\delta\mathcal{E} = T\delta S + \Phi_I \delta Q_I$, where $\Phi_I \equiv -\xi^a A_a^I |_{\mathcal{H}}$ is the electrostatic potential and $Q_I \equiv \int_r \frac{1}{(n-2)!} \frac{\partial \mathcal{L}}{\partial F_{ab}^I} \epsilon_{aba_1\cdots a_{n-2}} dx^{a_1} \wedge \cdots \wedge dx^{a_{n-2}}$ is charge. It is not difficult to find that in our case, $\Phi_I = -e_I$ and $Q_I = q_I$. The standard first law hence can be recovered by using these relations.

On the other hand, one integrates over another Cauchy surface $\mathcal{C}_{\text{AdS}}$ to get

$$\int_R \mathbf{Q}_{\text{AdS}}[t] = -\int_{\mathcal{C}_{\text{AdS}}} \xi^t \cdot \mathbf{L}(g_{\text{AdS}}). \tag{2.30}$$

Following the procedures made in last section, we variate above formula, and note

$$\delta \int_{\mathcal{H}} \mathbf{Q}[t] = \int_{r_H + \delta r} \mathbf{Q}[t] - \int_{r_H} \mathbf{Q}[t] = [(TS)' + e_I q_I + e_I q_I']\delta r, \tag{2.31}$$

and

$$\delta \int_{\mathcal{C}} (\xi^t \cdot \mathbf{L} + t \cdot \mathbf{B}) = -\delta F = \int_{r_H}^{r_H + \delta r} f dr = f(r_H)\delta r, \tag{2.32}$$

in the end we obtain our final result

$$(TS)' - \mathcal{E}' + e_I q_I' - (TS)' = \mathbf{f}_{BH} - \left[\left(\frac{g_{tt}^{BH}}{g_{tt}^{\text{AdS}}}\right)^{1/2}\right]_{r=\hat{R}} \mathbf{f}_{\text{AdS}}, \tag{2.33}$$

where $\mathbf{f}_{\text{BH}} = e_I q_I - f_{\text{BH}}$ and $\mathbf{f}_{\text{AdS}} = -f_{\text{AdS}}$ denote the entropy functions of black hole and AdS respectively. To obtain the correct entropy we should integrate the right-hand side of eq. (2.33) with appropriate integration region. A reasonable integration region is shown to be [37]

$$F = \int_{r_H}^{\hat{R}} \mathbf{f}_{\text{BH}} - \left[\left(\frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}}\right)^{1/2}\right]_{r=\hat{R}} \int_0^{\hat{R}} \mathbf{f}_{\text{AdS}}, \tag{2.34}$$

where $F = \mathcal{E} - TS + e_I q_I'$ is the free energy of the system. The entropy thus can be obtained by

$$S = -\frac{\partial F}{\partial T}.$$

In particular, the corresponding expression for pure dS (or AdS) is

$$TS = \int_0^{r_H} \mathbf{f}_{\text{dS}} dr, \tag{2.35}$$

where r_H is the event horizon of dS space. We can show the entropy of dS space in any R^2 gravity theories can be computed by[2]

$$S = S_0 + \gamma S_1 = \frac{1}{T_0} \int_0^{r_H^{(0)}} \left(\mathbf{f}_0 + \gamma \mathbf{f}_1^{(0)}\right) dr, \tag{2.36}$$

where superscript "(0)" denotes the variables computed by using the unperturbative metric, and $\mathbf{f}_0$ and $\mathbf{f}_1$ (including $\mathbf{f}_1^{(0)}$ and $\mathbf{f}_1^{(1)}$, see appendix A for detail) represent entropy function with and without higher derivative corrections respectively. Both $\mathbf{f}_0$ and T_0 are calculated by using the unperturbative metric. γ is a small quanta showing the coupling strength. This implies one can obtain the entropy of dS spacetime in any R^2 gravity theories without knowing the corrected metric. In our present paper, we will confirm this argument by taking several examples.

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[2]The details can be found in appendix A.

3. Entropy of D-dimensional de Sitter spacetime

In this section, we will give a simple example to demonstrate how our method works for non-supergavity spacetime. We may calculate entropy of pure dS spacetime by using the method discussed in the last section.

From the traditional point of view, the "entropy function" method may break down when we apply it to calculating the entropy of dS spacetime due to its non-extremality. Our analysis in the last section indicates we may end this embarrassed situation of this kind of space by extending the entropy function to the case including non-extremal spacetime. The main idea is to relate the entropy function with the black hole thermodynamics with the help of the entropy definition of Wald, whose definition is valid for any non-extremal black holes. By doing so, we obtained a formula by which one can compute the dS entropy, i.e.,

$$TS = \int_0^l \mathbf{f}_{\mathrm{dS}} dr, \tag{3.1}$$

where l denotes the cosmological radius. The Einstein-Hilbert action with a positive cosmological constant is given by

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-det\ g}(R - 2\Lambda), \tag{3.2}$$

where R is the Ricci scalar of the spacetime manifold, $\Lambda = (D-1)(D-2)/2l^2$ is the cosmological constant, and G_D is the D-dimensional Newton constant. The corresponding static metric is given by

$$ds^2 = -\left(1 - \frac{r^2}{l^2}\right)dt^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2. \tag{3.3}$$

Recalling the definition of f in (2.10), we obtain

$$\mathbf{f}_{\mathrm{dS}} = -\frac{(D-1)\pi^{\frac{D-3}{2}} r^{D-2}}{4G_D \Gamma(\frac{D-1}{2}) l^2}. \tag{3.4}$$

The temperature for D-dimensional dS spacetime is

$$T = -\frac{1}{2\pi l}, \tag{3.5}$$

Direct calculation of eq. (2.35) gives the entropy of dS spacetime

$$S_{\mathrm{dS}} = \frac{A}{4G_D} = \frac{l^{D-2} A_{D-2}}{4G_D}, \tag{3.6}$$

where $A_{D-2} = 2\pi^{(D-1)/2}/\Gamma((D-1)/2)$ is the area of the $(D-2)$ dimensional unit sphere. (3.6) is exactly the Bekenstein-Hawking entropy of dS spacetime. This indicates that our method discussed in section 2 works well for dS space.

4. Entropy with Gauss-Bonnet term in de Sitter space

In this section, we wish to confirm our argument made in section2, i.e., *one can obtain the entropy of dS spacetime in any R^2 gravity theories without knowing the corrected metric.* In particular, we shall consider a specific higher derivative correction to the action — the Gauss-Bonnet term. This term, which is generated from the heterotic and bosonic string theory low energy effective theory, is a natural correction term to the Einstein-Hilbert action (3.2). It corresponds to an additional term in the Lagrangian density of the form [38][3]

$$\mathcal{L}_{\text{GB}} = \frac{\alpha}{16\pi G_D} \left\{ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \right\}, \tag{4.1}$$

where α is the coupling constant with dimensions $(length)^2$. In particular, $\alpha = \alpha'/4$ in the low energy effective action of heterotic string theory. Then the action containing the Gauss-Bonnet term becomes [39]

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-det\ g}\ (R - 2\Lambda + \mathcal{L}_{\text{GB}}). \tag{4.2}$$

The corresponding equation of motion is [36]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{(D-1)(D-2)}{2l^2}g_{\mu\nu} =$$
$$\alpha \left\{ \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\text{GB}} - 2\left(R_{\mu\alpha\beta\gamma}R_\nu^{\alpha\beta\gamma} - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} - 2R_\mu^\alpha R_{\nu\alpha} + RR_{\mu\nu} \right) \right\} \tag{4.3}$$

4.1 Entropy function method

We wish to find the leading order correction in α to the entropy of the dS spacetime. Usually, before we do this, we should first obtain the corresponding solutions to the modified Einstein equation (4.3). This, however, turns out to be not so easy except for some special cases. Fortunately, the entropy function method provides us with an elegant way to calculate the entropy of higher derivative gravity without knowing the corrected metric. The Gauss-Bonnet term corresponding to the metric (3.3) is then given by

$$\mathcal{L}_{\text{GB}} = \frac{D(D-1)(D-2)(D-3)\alpha}{16\pi G_D l^4}. \tag{4.4}$$

The definition of f in (2.10) shows Lagrangian density with higher derivative correction gives unavoidably an extra term of f. In Gauss-Bonnet Einstein gravity, this term comes from the Gauss-Bonnet term as shown in (4.4). This in turn will change the expression of entropy function as defined in previous section. Direct computation shows the correction term to the entropy function (3.4) is

$$\mathbf{f}_{1GB} = -f_{\text{GB}} = -\int d^{D-2}x\sqrt{-detg}\mathcal{L}_{\text{GB}}$$
$$= -\frac{\alpha D(D-1)(D-2)(D-3)A_{D-2}}{16\pi G_D l^4}r^{D-2}. \tag{4.5}$$

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[3]What we should mention is that we require $D \geq 5$ in this case since the Gauss-Bonnet term is a topological invariant in four dimension.

Substituting this into entropy function equation (2.36), we can easily obtain the entropy generated by Gauss-Bonnet correction term

$$S_{1GB} = \frac{\int_0^l \mathbf{f}_{1GB} dr}{T} = \frac{\alpha D(D-2)(D-3)A_{D-2}}{8G_D} l^{D-4}. \tag{4.6}$$

Hereafter we do not distinguish T from T_0. Consequently, the entropy of dS space (near the horizon) including Gauss-Bonnet correction becomes

$$S = S_{\mathrm{dS}} + S_{1GB} = \frac{l^{D-2}A_{D-2}}{4G_D}\left(1 + \frac{\alpha D(D-2)(D-3)}{2l^2}\right) + \mathcal{O}(\alpha^2). \tag{4.7}$$

4.2 Wald's approach

In this subsection, we wish to check our result (4.7) obtained by entropy function method. One way is to use the Wald's approach. To do this, we have to find the corrected metric of the modified equation of motion (4.3) before we compute the corrected entropy of the Gauss-Bonnet gravity. Generally speaking, it is not easy to obtain the solution of this equation of motion. However, in some special cases, we can find the exact solution. Now we assume the metric to be of the form as discussed in [39]

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega_{D-2}^2, \tag{4.8}$$

where $\nu(r)$ and $\lambda(r)$ are functions of r only. The solution under this assumption is shown to be [39]

$$e^{2\nu} = e^{-2\lambda} = 1 + \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1 + \frac{4\tilde{\alpha}}{l^2}}\right), \tag{4.9}$$

where $\tilde{\alpha} = (D-3)(D-4)\alpha$. As shown in [39, 36], the branch with the "+" sign is unstable and the graviton is a ghost. Therefore we only consider the case with "−" sign in this paper. Consequently, the metric becomes

$$e^{2\nu} = e^{-2\lambda} = 1 + \frac{r^2}{2\tilde{\alpha}}\left(1 - \sqrt{1 + \frac{4\tilde{\alpha}}{l^2}}\right). \tag{4.10}$$

Since the horizon of this geometry occurs where $e^{2\nu} = 0$, we find the event horizon radius

$$\begin{aligned} r_H &= \left(\frac{2\tilde{\alpha}}{\sqrt{1 + 4\tilde{\alpha}/l^2} - 1}\right)^{1/2} \\ &= l + \frac{\tilde{\alpha}}{2l} + \mathcal{O}(\tilde{\alpha}^2). \end{aligned} \tag{4.11}$$

One may expect that we can obtain the Bekenstein-Hawking entropy by direct substituting (4.11) into the entropy-area formula $S_{\mathrm{BH}} = \frac{A}{4}$. This, however, has been proved to be no longer true by a lot of earlier investigations [40]. It was shown in [41] that S should take the form of a geometric expression evaluated at the event horizon. There are many ways to calculate the entropy of this spacetime: one way is to use the Wald's formula [42]; the other is related to the Euclidean entropy of the space as shown in [37, 43]. It also can

be obtained by assuming the spacetime satisfies the first law of thermodynamics as shown in [44]. As an example, we use Wald's approach to check our method obtained in the last subsection. Using the Wald's formula, we know that the entropy of a hole valid to any effective gravitational action including higher curvature interactions is given by [42]

$$S_{\text{Wald}} = \frac{1}{4G_D} \int_{\mathcal{H}} d^{D-2}x \sqrt{h} \left[1 + 2\alpha \tilde{R}(h) \right], \qquad (4.12)$$

where h_{ij} is the induced metric on the horizon,[4] and $\tilde{R}(h) = h^{ij}h^{kl}R_{ikjl}$ is the Ricci scalar calculated by using the induced metric h_{ij}. The entropy calculated in this way is proved to be [43]

$$\begin{aligned} S &= \frac{r_H^{D-2}A_{D-2}}{4G_D} \left(1 + \frac{2(D-2)\tilde{\alpha}}{(D-4)r_H^2} \right) \\ &= \frac{l^{D-2}A_{D-2}}{4G_D} \left(1 + \frac{D(D-2)\tilde{\alpha}}{2(D-4)l^2} \right) + \mathcal{O}(\tilde{\alpha}^2) \\ &= \frac{l^{D-2}A_{D-2}}{4G_D} \left(1 + \frac{D(D-2)(D-3)\alpha}{2l^2} \right) + \mathcal{O}(\alpha^2), \end{aligned} \qquad (4.13)$$

which is exactly the same as entropy (4.7) obtained by the entropy function method. Therefore we conclude that, compared with other methods (such as the entropy function method proposed by Sen and the Wald's approach), our method has many advantages. Roughly speaking, they are:

(i) Unlike the standard entropy function method, we do not need to extremize the entropy function, and also, we need not deform the metric into the near horizon geometry. The configurations of spacetime can be used directly without any change in calculating the entropy. The near horizon limit is also unnecessary.

(ii) Unlike the standard entropy function method, we do not need to introduce some constants into the metric, such as v_1 and v_2.

(iii) Unlike the Wald's approach or Euclidean approach [37], we do not need to know the corrected metric of higher derivative gravity for dS space.

5. Entropy with R^2 term in de Sitter space

In this section, we consider R^2 term which involves a correction proportional to $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. This term, which is the first term that one can add to the Einstein-Hilbert action, has been discussed in several previous works [37, 43]. It corresponds to an additional term in the Lagrangian density of the form [38]

$$\mathcal{L}_{R^2} = \frac{\alpha}{16\pi G_D} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \qquad (5.1)$$

---

[4]In present case, h_{ij} denotes the metric of sphere S^{D-2}.

where again α is the coupling constant with dimensions $(length)^2$, and $\alpha = \alpha'/4$ in the low energy effective action of heterotic string theory. The action containing this term becomes

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-det\ g}\, (R - 2\Lambda + \mathcal{L}_{R^2})\,. \tag{5.2}$$

The corresponding equation of motion is [37]

$$R_{\mu\nu} - \frac{Rg_{\mu\nu}}{2} + \frac{(D-1)(D-2)}{2l^2}g_{\mu\nu} = \frac{\alpha}{2}g_{\mu\nu}\mathcal{L}_{R^2} - 2\alpha\bigg(R_{\mu\alpha\beta\gamma}R_\nu^{\alpha\beta\gamma} + 2R^{\alpha\beta}R_{\mu\alpha\nu\beta}$$
$$-2R_\mu^\alpha R_{\nu\alpha} + 2\Box R_{\mu\nu} - \frac{1}{2}(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu)R\bigg)\,. \tag{5.3}$$

5.1 Entropy function method

In this subsection, we evaluate the R^2 correction to the entropy of the dS spacetime. As mentioned in the last section, we do not need to know the modified metric before compute the corrected entropy by using the entropy function method. Therefore we start with calculating the R^2 term corresponding to the uncorrected metric (3.3)

$$\mathcal{L}_{R^2} = \frac{D(D-1)\alpha}{8\pi G_D l^4}\,. \tag{5.4}$$

The definition of f in (2.10) shows Lagrangian density with higher derivative correction gives unavoidably an extra term of f which comes from the R^2 term as shown in (5.4). This in turn will change the expression of entropy function as defined in previous section. Direct computation shows the correction term to the entropy function (3.4) is

$$\mathbf{f}_{1R^2} = -\frac{\alpha D(D-1)A_{D-2}}{8\pi G_D l^4}r^{D-2}\,. \tag{5.5}$$

Substituting this into eq. (2.36), we can easily obtain the entropy function generated by R^2 correction term

$$S_{1R^2} = \frac{\alpha D A_{D-2}}{4G_D}l^{D-4}\,. \tag{5.6}$$

Consequently, the entropy of dS space including R^2 correction becomes

$$S = S_{\rm dS} + S_{1R^2} = \frac{l^{D-2}A_{D-2}}{4G_D}\left(1 + \frac{\alpha D}{l^2}\right) + \mathcal{O}(\alpha^2)\,. \tag{5.7}$$

5.2 Wald's approach

In last subsection, we have calculated the corrected entropy for dS space by using entropy function method. To confirm our result, we now appeal to Wald's approach. To do this, we should first find the corrected metric of the modified equation of motion (5.3). Generally speaking, it is not easy to obtain the solution of this equation of motion. However, in some special case, we can find a perturbative metric solution. Now we assume the metric to be of the form

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega_{D-2}^2\,, \tag{5.8}$$

where ν and λ are functions of r only, and have the form

$$
\begin{aligned}
e^{2\nu} &= e^{2\nu_0}(1 + \alpha\epsilon(r)), \\
e^{2\lambda} &= e^{-2\nu_0}(1 + \alpha\mu(r)),
\end{aligned}
\tag{5.9}
$$

where $e^{2\nu_0} = 1 - r^2/l^2$, $\epsilon(r)$ and $\mu(r)$ are some undetermined functions of r, which can be obtained by evaluating the equation of motion (5.3) perturbatively. The result reads

$$
\epsilon(r) = -\mu(r) = e^{-2\nu_0}\frac{2(D-4)r^2}{(D-2)l^4}.
\tag{5.10}
$$

Consequently, the metric becomes

$$
e^{2\nu} = e^{-2\lambda} = 1 - \frac{r^2}{l^2} + \frac{2\alpha(D-4)r^2}{(D-2)l^4},
\tag{5.11}
$$

Since the horizon of this geometry occurs where $e^{2\nu} = 0$, we see

$$
\begin{aligned}
r_H &= \frac{l}{\sqrt{1 - \frac{2(D-4)\alpha}{(D-2)l^2}}} \\
&= l + \frac{(D-4)\alpha}{(D-2)l} + \mathcal{O}(\alpha^2).
\end{aligned}
\tag{5.12}
$$

Using the Wald's formula, we know that the entropy of a hole valid to any effective gravitational action including higher curvature interactions is given by [42]

$$
S_{\text{Wald}} = \frac{1}{4G_D}\int_{\mathcal{H}} d^{D-2}x\sqrt{h}\left[1 + 2\alpha\left(R - 2h^{ij}R_{ij} + \tilde{R}(h)\right)\right],
\tag{5.13}
$$

where again $\tilde{R}(h) = h^{ij}h^{jk}R_{ikjl}$. Direct computation shows

$$
\tilde{R} = \frac{(D-2)(D-3)}{r_H^2} + \mathcal{O}(\alpha)
\tag{5.14}
$$

$$
h^{ij}R_{ij} = \frac{(D-1)(D-2)}{l^2} + \mathcal{O}(\alpha)
\tag{5.15}
$$

$$
R = \frac{D(D-1)}{l^2} + \mathcal{O}(\alpha).
\tag{5.16}
$$

Therefore, the entropy calculated in this way is

$$
\begin{aligned}
S &= \frac{r_H^{D-2}A_{D-2}}{4G_D}\left[1 + 2\alpha\left\{\frac{(D-2)(D-3)}{r_H^2} - \frac{(D-1)(D-4)}{l^2}\right\}\right] \\
&= \frac{l^{D-2}A_{D-2}}{4G_D}\left[1 + \frac{(D-4)\alpha}{l^2}\right]\left(1 + \frac{4\alpha}{l^2}\right) + \mathcal{O}(\alpha^2) \\
&= \frac{l^{D-2}A_{D-2}}{4G_D}\left(1 + \frac{D\alpha}{l^2}\right) + \mathcal{O}(\alpha^2),
\end{aligned}
\tag{5.17}
$$

which is exactly the same as entropy (5.7) obtained by the entropy function method.

6. Black hole Entropy in 5−dimensional (anti-) de Sitter space

In previous sections, we investigate the entropy function for pure dS space by using a powerful method developed in [35]. We showed that entropy obtained by this method agrees with the one computed by other approaches (say, Wald's approach) very well. Moreover, for that case we can obtain the correct entropy with higher derivative corrections without knowing the modified metric. A question is that what will happen if there is a black hole in the dS (or AdS) background. How to calculate the entropy of this case especially when there are higher derivative corrections? In this section, we try to answer this question by applying our method to a 5-dimensional black hole in the Type IIB string theory. We will show that, although it is no longer possible to obtain the entropy without knowing the modified metric when higher curvature corrections are included, our method is still applicable to this case.

6.1 Entropy of Schwarzschild-AdS and Schwarzschild-dS black hole

The 5−dimensional IIB superstring effective action which is obtained by compactifying the ten dimensional action on the S^5 is [45][5]

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-\det g}\left[R - 2\Lambda\right].$$ (6.1)

We first pay our attention to the Schwarzschild-AdS black hole. In the throat approximation, $r \ll l$, a solution corresponding to the AdS black hole is [45, 46]

$$ds^2 = \frac{r^2}{l_{\text{AdS}}^2}\left[-\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \frac{l_{\text{AdS}}^2}{r^2}\left(1 - \frac{r_0^4}{r^4}\right)^{-1}dr^2$$ (6.2)

The Lagrangian corresponding to this metric reads

$$\mathcal{L} = -\frac{1}{2\pi G_5 l_{\text{AdS}}^2}.$$ (6.3)

Therefore the entropy function can be obtained by its definition

$$\mathbf{f}_{\text{BH}} = -f_{\text{BH}} = -\int d^3\vec{x}\sqrt{-\det g}\mathcal{L} = \frac{V_3 r^3}{2\pi G_5 l_{\text{AdS}}^5},$$ (6.4)

where $V_3 = \int d^3\vec{x}$ is the volume. On the other hand, the entropy function for pure AdS background is given by

$$\mathbf{f}_{\text{AdS}} = -\int d^3\vec{x}\sqrt{-\det g_{\text{AdS}}}\mathcal{L}_{\text{AdS}} = \frac{V_3 r^3}{2\pi G_5 l_{\text{AdS}}^5},$$ (6.5)

which is the same as the one of the black hole. Therefore from eq. (2.34) we obtain the free energy of the system (Note that in present case, $r_+ = r_0$)

$$F = \int_{r_0}^{\hat{R}} \mathbf{f}_{\text{BH}} - \left[\left(\frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}}\right)^{1/2}\right]_{r=\hat{R}} \int_0^{\hat{R}} \mathbf{f}_{\text{AdS}},$$ (6.6)

$$\simeq -\frac{V_3 r_0^4}{16\pi G_5 l_{\text{AdS}}^5}.$$ (6.7)

---

[5]We have set the dilaton to a constant and we omit it in this paper.

According to the definition (2.28), the ADM energy in this case is given by [37]

$$\mathcal{E} = \int_{\hat{R}} \mathbf{Q}[t] - \int_{\hat{R}} \mathbf{Q}_{\text{AdS}}[\tilde{t}] \tag{6.8}$$

$$= \int_{\hat{R}} dS_{ab} \sqrt{-det\ g} Q^{ab} - \left[\left(\frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}} \right)^{1/2} \right]_{r=\hat{R}} \int_{\hat{R}} dS_{ab} \sqrt{-det\ g_{\text{AdS}}} Q_{\text{AdS}}^{ab}, \tag{6.9}$$

where

$$Q^{ab} = \frac{\kappa}{8\pi G_5},$$

and $\kappa = \left(\frac{r}{l^2} + \frac{r_0^4}{l^2 r^3} \right) \big|_{r=r_0}$ is the surface gravity of the black hole. In the end we obtain

$$\mathcal{E} = \frac{3V_3 r_0^4}{16\pi G_5 l_{\text{AdS}}^5}. \tag{6.10}$$

The temperature at the horizon for this black hole AdS spacetime is

$$T = \frac{r_0}{\pi l_{\text{AdS}}^2}, \tag{6.11}$$

Therefore using the relation

$$TS = \mathcal{E} - F, \tag{6.12}$$

we obtain the entropy of Schwarzschild-AdS black hole

$$S = \frac{V_3 r_0^3}{4G_5 l_{\text{AdS}}^3}. \tag{6.13}$$

This is exactly the Bekenstein-Hawking entropy for this black hole [37].

It is not difficult to check that following the above procedure, we can obtain the entropy of the Schwarzschild-dS black hole. The result is proved to be the same form as the one for the Schwarzschild-AdS black hole, i.e.,

$$S = \frac{V_3 r_0^3}{4G_5 l_{\text{dS}}^3}. \tag{6.14}$$

6.2 R^4 correction to the entropy

In this subsection, due to the AdS/CFT (or dS/CFT) correspondence, we wish to study the corrections that appear in the Type IIB string theory. These corrections to supergravity action come from string theory tree-level scattering amplitude computations. R^4 term is the first higher order gravity correction terms in IIB action. We may focus on this correction term. It corresponds to an additional term in the action (6.1)

$$I = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-det\ g} \left[R - 2\Lambda + \gamma \mathcal{L}_{R^4} \right], \tag{6.15}$$

where $\gamma = \frac{1}{8} \zeta(3)(\alpha')^3$, and the correction term in the Lagrangian density is of the form

$$\mathcal{L}_{R^4} = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk}, \tag{6.16}$$

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where C_{pqmn} is the Weyl tensor.

First we calculate the entropy of Schwarzschild-AdS black hole with R^4 correction. Using the metric (6.2) one can compute $\mathcal{L}_{R^4}$

$$\mathcal{L}_{R^4} = \frac{180r_0^{16}}{r^{16}l_{\text{AdS}}^8}. \tag{6.17}$$

In this case, the modified metric reads [45]

$$ds^2 = -e^{2\lambda}dt^2 + e^{2\nu}dr^2 + \frac{r^2}{l_{\text{AdS}}^2}d\vec{x}^2, \tag{6.18}$$

where

$$e^{2\lambda} = \frac{r^2}{l_{\text{AdS}}^2}\left(1 - \frac{r_0^4}{r^4}\right)\left[1 - \frac{15\gamma r_0^4}{l_{\text{AdS}}^6 r^{12}}(5r^8 + 5r^4 r_0^4 - 3r_0^8)\right], \tag{6.19}$$

$$e^{2\nu} = \frac{r^2}{l_{\text{AdS}}^2}\left(1 - \frac{r_0^4}{r^4}\right)^{-1}\left[1 + \frac{15\gamma r_0^4}{l_{\text{AdS}}^6 r^{12}}(5r^8 + 5r^4 r_0^4 - 19r_0^8)\right]. \tag{6.20}$$

Therefore the correction term to the entropy function reads

$$\mathbf{f}_{1BH} = -\frac{30\gamma V_3}{4\pi G_5}\frac{r_0^{12}}{r^9 l_{\text{AdS}}^{11}}\left(8 + \frac{3r_0^4}{r^4}\right). \tag{6.21}$$

The R^4 correction does not change the metric of the AdS background,[6] so the correction of AdS entropy function is vanishing, i.e., $\mathbf{f}_{1AdS} = 0$. After letting $R \to \infty$ we find the expression of the free energy

$$F = \int_{r_0}^{\hat{R}}\mathbf{f}_{\text{BH}} - \left[\left(\frac{g_{tt}^{\text{BH}}}{g_{tt}^{\text{AdS}}}\right)^{1/2}\right]_{r=\hat{R}}\int_0^{\hat{R}}\mathbf{f}_{\text{AdS}}, \tag{6.22}$$

$$\simeq -\frac{V_3 r_0^4}{16\pi G_5 l_{\text{AdS}}^5}\left(1 + \frac{75\gamma}{l_{\text{AdS}}^6}\right). \tag{6.23}$$

On the other hand, the temperature of the hole is

$$T = \frac{\kappa}{4\pi} = \left[\sqrt{g^{rr}}\frac{d}{dr}\sqrt{g_{tt}}\right]_{r=r_0} = \frac{r_0}{\pi l_{\text{AdS}}^2}\left(1 + \frac{15\gamma}{l_{\text{AdS}}^6}\right). \tag{6.24}$$

---

[6]Generally speaking, to obtain the modified metric, one should first variate the action and obtain the equations of motion, which take the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \gamma T_{\mu\nu}^{\text{eff}},$$

where $T_{\mu\nu}^{\text{eff}}$ are constructed by Weyl tensor in present case. The modified metric is then given by solving these equations of motion. To the first order, we substitute the unperturbative metric into R.H.S of the equations. For pure AdS and dS metric, it is not difficult to check that all components of the Weyl tensor and hence $T_{\mu\nu}^{\text{eff}}$ are vanishing. This implies the AdS metric is unchanged when R^4 correction is included. This is also confirmed in [45], where they calculate the black hole entropy using Euclidean approach and state that the action of AdS background I_0 is unchanged when R^4 is considered.

Direct computation shows the corrected entropy of Schwarzschild-AdS black hole with R^4 corrections is

$$S = -\frac{\partial F}{\partial T} \tag{6.25}$$

$$= \frac{V_3 r_0^3}{4G_5 l_{\text{AdS}}^3} \left(1 + \frac{60\gamma}{l_{\text{AdS}}^6}\right). \tag{6.26}$$

This is exactly the entropy of this case as shown in [37].

Again one can obtain the entropy of the Schwarzschild-dS black hole using the same procedures above. Compared with the result for the case without higher corrections, the entropy of the Schwarzschild-dS black hole with R^4 corrections is different from the entropy of the case for Schwarzschild-AdS black hole. In present case, the entropy is

$$S = \frac{V_3 r_0^3}{4G_5 l_{\text{dS}}^3} \left(1 - \frac{60\gamma}{l_{\text{dS}}^6}\right). \tag{6.27}$$

7. Conclusions

In summary, we have discussed higher order corrections to black hole entropy in dS and AdS spaces. We started from pure dS and AdS spaces and we have found that even when the higher curvature corrections are included, the corrected entropy for dS space can still be calculated by using the equation $(TS)' = -\mathbf{f}$. The entropy function method and Wald's approach in calculating the entropies of dS and AdS spaces agree with each other. As we have described, the agreement between these two approaches can be understood, from black hole thermodynamics [35].

Although we do not know the general near horizon geometry of non-extremal black holes in higher derivative gravity theory, our results demonstrate that extending the entropy function formalism to pure dS (AdS) space is safe. However, we do not find an universal way to generalize the entropy function to more complicated cases, such as non-extremal and rotating black holes. For these cases one need first find that the symmetries of the near horizon geometry for non-extremal and rotating black holes in higher derivative gravity theory.

Note that the method used in this work is quite different from the standard *entropy function method* used in ref. [1, 16] in that the entropy is obtained without extremizing the entropy function. Since there are no free parameters, such as v_i and u_s, whose values should be determined by extremizing the entropy function, the extremizing process is unnecessary. And also, the near horizon limit is not done for (anti-) de Sitter metric. The reason is because that for the dS and AdS metric, the metric configuration is not so complicate as stringy black hole metric.

A. de Sitter entropy in R^2 gravity theories

In this appendix we consider the entropy for pure de Sitter spacetime with "higher derivative" gravity. The entropy of this case has been shown to be given by (2.35)

$$TS = \int_0^{r_H} \mathbf{f}_{\mathrm{dS}} dr, \tag{A.1}$$

where r_H is the event horizon of dS space. For R^2-gravity with cosmological constant, the general action is given by

$$S = \int d^D x \sqrt{-g} \left\{ R - 2\Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right\}. \tag{A.2}$$

Variation over the metric $g_{\mu\nu}$ yields the equations of motion as shown in [47]

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \left(aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma} + R - 2\Lambda \right) \tag{A.3}$$

$$- a \left(-2RR^{\mu\nu} + \nabla^\mu\nabla^\nu R + \nabla^\nu\nabla^\mu R - 2g^{\mu\nu}\nabla_\rho\nabla^\rho R \right)$$

$$- b \left\{ \frac{1}{2} \left(\nabla^\mu\nabla^\nu R + \nabla^\nu\nabla^\mu R \right) - 2R^{\mu\rho\nu\sigma}R_{\rho\sigma} - \Box R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}\Box R \right\}$$

$$- c \left(-2R^{\mu\rho\sigma\tau}R^\nu{}_{\rho\sigma\tau} - 4\Box R^{\mu\nu} + \nabla^\mu\nabla^\nu R + \nabla^\nu\nabla^\mu R \right.$$

$$\left. -4R^{\mu\rho\nu\sigma}R_{\rho\sigma} + 4R^\mu{}_\rho R^{\nu\rho} \right) = 0.$$

We may expect that the modified metric of dS space takes the form

$$ds^2 = - \left[1 - \frac{r^2}{l^2}(1+C) \right] dt^2 + \left[1 - \frac{r^2}{l^2}(1+C) \right]^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \tag{A.4}$$

where C is a constant which should be determined later. With this metric ansatz, we obtain an equation of C by substituting the metric into eqs. (A.3)

$$[D(D-1)a + (D-1)b + 2c](D-4)C^2$$

$$+\{2[D(D-1)a + (D-1)b + 2c](D-4) + (D-2)l^2\}C$$

$$+[D(D-1)a + (D-1)b + 2c](D-4) = 0. \tag{A.5}$$

Straightforward computation gives

$$C = -1 - \frac{(D-2)l^2}{2[D(D-1)a + (D-1)b + 2c](D-4)} \cdot$$

$$\left\{ 1 \mp \sqrt{1 + \frac{4[D(D-1)a + (D-1)b + 2c](D-4)}{(D-2)l^2}} \right\}. \tag{A.6}$$

In particular, as we consider the Gauss-Bonnet combination, i.e., $a = c, b = -4c$, we have

$$C = -1 - \frac{l^2}{2(D-3)(D-4)c} \left\{ 1 \mp \sqrt{1 + \frac{4(D-3)(D-4)c}{l^2}} \right\}, \tag{A.7}$$

which agrees with the result obtained in [39].

Now it is safe to claim that *dS space in R^2 gravity theories has a modified metric in the form of (A.4)*. In the following we will show that one can obtain the dS entropy in any gravity theories with modified metric in the form of (A.4) without knowing the corrected metric. In order to have a clear picture, we rewrite the metric (A.4) as

$$ds^2 = -\left[1 - \frac{r^2}{l^2}(1 + \gamma\tilde{C})\right] dt^2 + \left[1 - \frac{r^2}{l^2}(1 + \gamma\tilde{C})\right]^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \qquad \text{(A.8)}$$

where γ is the coupling strength, denoting a, b, c for R^2 gravity theory, and $\tilde{C}$ is any constant. In any higher derivative gravity theories, it is safe to expand T, S and $\mathbf{f}$ in the following way

$$T = T_0 + \gamma T_1 + \mathcal{O}(\gamma^2), \qquad \text{(A.9)}$$

$$S = S_0 + \gamma S_1 + \mathcal{O}(\gamma^2), \qquad \text{(A.10)}$$

$$\mathbf{f} = \mathbf{f}_0 + \gamma \mathbf{f}_1 + \mathcal{O}(\gamma^2). \qquad \text{(A.11)}$$

So from (2.35) we have

$$T_0 S_0 + \gamma(T_0 S_1 + T_1 S_0) = \int_0^{r_H^{(0)}} \mathbf{f}_0 dr + \int_{r_H^{(0)}}^{r_H^{(0)} + \gamma r_H^{(1)}} \mathbf{f}_0 dr + \gamma \int_0^{r_H^{(0)}} \mathbf{f}_1 dr, \qquad \text{(A.12)}$$

where $r_H^{(0)}$ corresponds to the horizon of dS space in the case without the higher derivative, while $r_H^{(1)}$ is the first order correction to the horizon when higher derivative is considered. This implies the horizon for higher derivative gravity is $r_H = r_H^{(0)} + \gamma r_H^{(1)} + \mathcal{O}(\gamma^2)$. In particular, if the modified metric has the form as shown in (A.8), we have $r_H^{(0)} = l$ and $r_H^{(1)} = -\frac{l\tilde{C}}{2}$. Then T_1 can be evaluated as

$$T_1 = \frac{r_H^{(1)}}{2\pi l^2}. \qquad \text{(A.13)}$$

Note that $\mathbf{f}_1$ includes two parts: one comes from the Hilbert-Einstein action (i.e., the Ricci scalar) evaluated by the modified metric (A.8), here we denote it by $\mathbf{f}_1^{(1)}$; the other corresponds to the R^2 gravity term calculated by unperturbative metric, we use $\mathbf{f}_1^{(0)}$ to distinguish it from the first one.

For de Sitter spacetime, we have

$$S_0 = \frac{l^{D-2}A_{D-2}}{4G_D}, \quad \mathbf{f}_0 = -\frac{(D-1)A_{D-2}r^{D-2}}{8\pi G_D l^2}, \quad \mathbf{f}_1^{(1)} = -\frac{D(D-1)\tilde{C}A_{D-2}r^{D-2}}{16\pi G_D l^2}.$$

It is not difficult to show

$$\gamma T_1 S_0 = \int_{r_H^{(0)}}^{r_H^{(0)} + \gamma r_H^{(1)}} \mathbf{f}_0 dr + \gamma \int_0^{r_H^{(0)}} \mathbf{f}_1^{(1)} dr. \qquad \text{(A.14)}$$

Therefore eq. (3.6) shows that the entropy for dS space in higher derivative gravity theory can be computed by

$$S = S_0 + \gamma S_1 = \frac{1}{T_0} \left\{ \int_0^{r_H^{(0)}} \mathbf{f}_0 dr + \gamma \int_0^{r_H^{(0)}} \mathbf{f}_1^{(0)} dr \right\}. \qquad \text{(A.15)}$$

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